

# **Ellipses: A Lesson for Pre-Calculus Classes**

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This lesson will continue to build on a discussion of conic sections, moving away from circles and into ellipses. This discussion will take place via a conversation on the United Arab Emirates Space Agency and its Hope Probe Mars Mission. By introducing the UAE Space Agency and its goals for Mars exploration, students will discuss elliptical orbitals, their shapes, how to model those shapes, and the information that can be gleaned. Students will also be given the opportunity to review exponential growth and how to model it in context.

#### Standards

## North Carolina Pre-Calculus Standard Course of Study

(http://www.ncpublicschools.org/docs/curriculum/mathematics/scos/current/pre-cal.pdf)

<u>Competency Goal 1</u>: The learner will describe geometric figures in the coordinate plane geometrically.

*Objective 1.02*: Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.

- a) Solve using tables, graphs, and algebraic properties.
- b) Interpret the constants and coefficients in the context of the problem.

<u>Competency Goal 2</u>: The learner will use relations and functions to solve problems.

*Objective 2.01*: Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.

- a) Solve using graphs and algebraic properties.
- b) Interpret the constants, coefficients, and bases in the context of the problem.

#### Instruction

Students will review the equation of a circle,  $(x - h)^2 + (y - k)^2 = r^2$ , as well as the characteristics gleaned from it, namely the center, (h, k), and the radius, r. Students will then be introduced to the United Arab Emirates Space Agency, its history, its goals, and the missions that are currently being planned, with a specific emphasis on the Hope Probe mission.

From our visit with His Excellency Dr. Mohamed Al Junaibi (Executive Director of the Space Sector) and Naser Al Hammadi (Head of Strategic Partnerships Division at the UAE Space Agency), we learned:

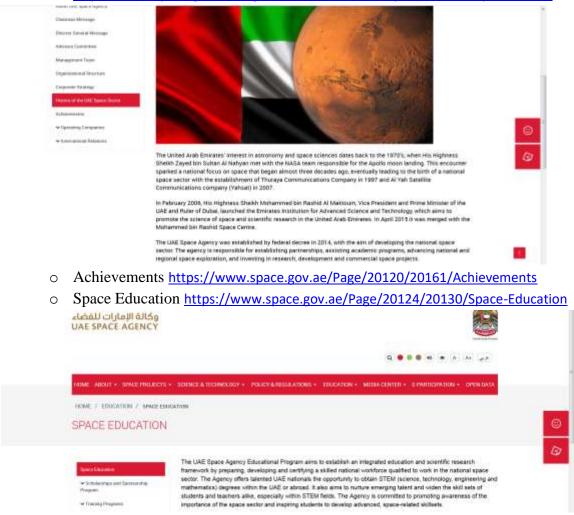
- Economic diversification is one of the impetuses of the creation of the UAE Space Agency – 70% non-oil, including space (which is connecting back into alternative energy and aerospace designs)
  - Designed to help create global citizens, centered around all-peace missions
- Why does the UAE want to go to space?

- Strategic goals: security in a turbulent region, the UAE is the 2<sup>nd</sup> strongest economy and aims to be a leader in scientific innovation and bringing stability to the region
- Scientific goals: ideas and inventions drive innovation, both in the space realm and in aerospace initiatives and advances in technology applicable in daily life
- Economic goals: to encourage technological innovations and start-ups that can be utilized across economic areas
- Social goals: navigation and urban planning designs to make lives better
- Aspirations, such as the Mars Mission, drive solutions to challenges that are presented, and those solutions are ten applied to other fields and areas
- History:
  - 1970's UAE founded, Sheikh Zayed sees Apollo missions and begins to think about how to utilized space
  - o 1980's Universities were started and scientific research began
  - 1990's Thuraya Telecommunications Company developed in Dubai
  - 2000's Mohamed Bin Rashid Space Center and Yahsat Satellite Communications Company established
  - $\circ$  2014 UAE space agency established
    - It governs the space sector
    - 3 space R&D centers that exceed \$6 billion in investment
    - Develop human capital and work with outside agencies and countries, including India, USA, China, Japan, Russia, France, Korea, and Kazakhstan
- The UAE is part of 4 major UN treaties regarding space and are an active member of COPUOUS, ISECG, GEO, The Charter, IPDA, IAF, and more
  - International partnerships and cooperation through treaties with ~20 different countries
- 2020 will be The Year of Space
  - Mars Mission 'The Hope' Want to launch probe to orbit Mars by 2020, and it will arrive there by 2021
  - UAE also has plans for a Mars Colony in 2117
    - Will create Mars Science City in Dubai
    - Focus: education; food, water, and energy challenges (both in UAE and on Mars); labs to include museums and provide entertainment
    - Will be the largest science city in the world (1.9 million ft<sup>2</sup>)
    - Researchers from all around the world will come to do experiments and do simulations of life on other planets
  - Astronaut program:
    - 2 Emirati citizens currently training to go to the International Space Station in Sept 2019 – one is a fighter pilot and one is a communications engineer with PhD in cyber security
    - Had 4,000 applicants, ~34% of whom were female
      - Were pilots, engineers, doctors, scientists, physicists, and mathematicians all were considered through interviews, screenings, and physical testing
      - Still want their first female astronaut, hope to select more as space program grows

- Trained in the US and some in Europe
- 4 applicants made it through the first rounds of cuts, then these final two were selected and 1 will fly in September – the other will go up in a mission next year
- Holding internal events
  - "Think Science" happens every Sunday in an emirate
  - Winter and summer camps within the UAE and in other countries, some of which have been organized with the USA and with Australia
  - Mobile planetariums go to schools
    - Educational awareness is huge, especially with a push in STEM education (12-30% of educational coursework) working to make STEM more interesting to youth
    - Space camps host astronauts into schools and universities
- Overall, the UAE Space Agency and its work are meant to fulfill the dreams of Sheikh Zayed, develop the UAE as a regional and global leader in space, and be a stabilizing influence in the Middle East

Further information can be found at this website -- https://www.space.gov.ae/.

- Students can either explore this website independently for 10-15 minutes, or the teacher can steer students toward topics of interest.
- Pages of interest:
  - History of the UAE Space Sector https://www.space.gov.ae/Page/20120/20158/History-of-the-UAE-Space-Sector



## • NASA Internship Program <u>https://www.space.gov.ae/Page/20124/20239/Nasa-</u>

NASA I <sup>2</sup> International Internation Program        V Solute those and Sponsorition      Program Announcement        Program      WOULD YOU LIKE TO INTERN AT NASA?        • Taxing Program      The UAE Space Agency (UAESA) in collaborations with The National Aeropauties and Space Topy Qualified UAE states to interna abrophic students from the United States. The WA	
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• About UAE Space Agency <u>https://www.space.gov.ae/Page/20120/20230/About-UAE-Space-Agency</u>

#### ABOUT UAE SPACE AGENCY The UAE Space Agency is a federal agency thet was created under Federal Law by Decree No. 1 of 2014. The space sector includes all projects, activities and programs related to outer space Chapters Message The decree stipulates that the UAE Space Agency works in line with the Council of Ministers and has an independent legal position, enjoying financial and administrative independence as well as the legal capacity necessary to direct all Unictor Goranal Manage activities that will ensure the achievement of its objectives. Amoug Convenies The law defined that the main head quarters of the Agency are to be in Abu Dhabi, and the Agency is to have a branch in Dubal. The Board of Directors may establish branches or other offices within and outside the state. Management Toppe Organization Structure The main aims of the UAE Space Agency are: Corporate Unintegy . To organise, regulate and support the space sector in the country and to enhance its position in this area. Henry of the LWE Space Sector · Encourage the development and use of space science and technology in the country and advance within the industry Addressed + The establishment of international partnerships in the space sector and to enhance the role of the state and its without in the second second position in the space sector. · Contribute to the diversification of the national economy through the space sector w international Relations · Raise awareness of the importance of the space technologies, enhance national capabilities and encourage peaceful application of space research. Jurisdiction The Agency is focused on the development of policies, strategies and plans related to the space sector that are approved by the Council of Ministers. It will provide advice and guidance for national space programs, work to resolve the challenges faced, and support research and studies for theoretical and applied areas within space. The Agency will also be involved in documenting and discerninating information, working on the development of human resources and support educational activities in the space sector and attract national talem into the industry. The Agency will Space Missions https://www.space.gov.ae/Page/20121/20220/Space-Missions 0 وكالة الإمارات للفضاء UAE SPACE AGENCY

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Name Manada Page Pycke Manalae URE Mice Sacrice Challenge	A nation of determined and ambitious vision, the UAE aims to play a leading role in the global space sector by 2021. In order to develop the UAE's space sector, the mission's carried out by the UAE Space Agency will advance science, technology and innovation throughout space-related fields. Such advances will allow the UAE to be a consender in the space race, developing Emirati capabilities in collaboration with local and international partners to the benefit of humankind. The Mars Exploration Mission is the first of many upcoming exploration missions to build an advanced and usstainable space program.	G

 MeznSat <u>https://www.space.gov.ae/Page/20121/20223/MeznSat</u> (spend particular time here discussing the impacts of climate change on resource management and scarcity – namely water – in the UAE and the connections of the issues with the Space Agency agenda)





Member is a new satelline interest and funded by the UAE Space Agency and in permership with lobelite University and the American University of Res Al Khaimah (AURAA). MemberShat is a 3U CuberShi to be developed, built and tested primarily by university students to detect Greenhouse Gas (DHG) concentrations. The project aims to offer the UAE space industry with qualified well-traced graduates through Sandta or experience. In addition, MemSat upens windows for advenced space-oriented research relevant to the UAE.

Planned to learnch end-of-2019, MeanGat will be developed by assidents, primarily from AURAK and Schalfle University, but also from other partnering educational institutions, who will await themselves of the world-class facilities evailable in Yahlar Spece Lab at Kihaifle University. The satellite will be launched from a site in Japan operated by the Japan Ammpace Exploration Approx. Once in orbit, the team of students will then monitor, process, and analyze the date from a ground station in the UAE.

Climate change has widely been attributed to the increase in GHGs in the atmosphere as a result of human activities. The impacts of climate change are espected to include shortage of water quantity and quality in most aid and semiarid areas, and low agricultural productivity throughout the tripics and subtropics, accomparied by damage to ecosystems and biodiversity in these areas, and changes in forests and other ecosystems. The State of the Environment (So6) Report for Abu Dhabi highlighted key vulnerabilities associated with climate change, principally see-level rise coastal flooding, increased satisfy of coastal aquifers, impacts on the marine environment, heat stress, built environment impacts, more extreme weather events (floods, droughts, etc.), increased risk of dust storms; and risk from altorne contaminants (e.g. pesticides).

Carbon Dioxide and Methane are the two most prevalent Greenhouse gases. Both emissions (methane and carbon dioxide) have to be addressed and monitored in order to effectively reduce the impact of climate change.

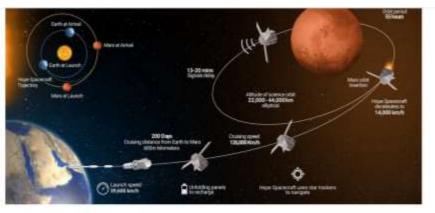
As a result, the primary scientific objective of this project is almed at exploring the performance of sensing in the shortwave infrared (SWIR) region (1000–1650 nm) to detect the levels of CH4, CD2, and H2O in order to derive the atmospheric concentrations of important GHGa. This mission follows the previous missions like Car0-2, SathyabhamaSat, etc.

Algal blooms in the Anabian Gulf can have a significant detrimental effect on the economy and health in the UAE. The Arabian Gulf hosts some of the largest desaination plants in the world and most of them are located along the UAE coastline. They account for a big percemage of the potable water consumed by UAE residems and the desalination capacity of the plants is constantly increasing. However, a recent surge in algal blooms in the Arabian Gulf is threatening the optimal operation of these desalination plants, which in turn will affect water availability in the UAE. Efficient monitoring and early sensing of algal blooms is required to efficiently manage the operations of the desalination plants. Algal blooms are also threatening the water quality in the coastal areas thus becoming a health hazard.

Therefore, the secondary/tentative scientific objective is to predict algal blooms in advance. The performance of sensing in the shortwave infrared (SWIR) region (1000–1050 nm) in combination with the RGE camera will be explored to estimate the concentration of total suspended matter (as a proxy for nutrients in water) in the coastal waters of the Arbian Guil to predict an algal bloom in advance, to facilitate precautionary measures.

Hope Probe <u>https://www.space.gov.ae/Page/20121/20167/Hope-Probe</u> (highlight this one last, as it will serve as the anchor of the lesson and introduction to ellipses – utilizing the link at the bottom of the page will redirect to the Emirates Mars Mission page -- <u>http://emiratesmarsmission.ae/mission-journey</u> -- and will provide additional information on the project)

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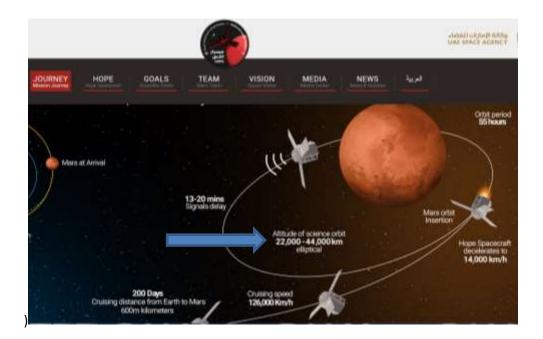


The United Arab Emirates has officially entered the global space exploration race. This came after His Highness Sheikh Khalifa bin Zayed al Nahyan, President of the UAE, issued a decree to establish the UAE Space Agency, and began working on sending the first Arab and Islamic probe to Mars, a project named "Hope Probe". This makes the UAE one of only nine countries working to explore Mars. The probe will start its journey in 2020, and is scheduled to arrive to Mars by 2021, in celebration of the 50th anniversary of the union of the UAE. The planning, management and implementation of the probe project are implemented by an Emirati team, whose members depend on their skills and diligence to acquire all space exploration-related knowledge and applying it. The program is fully funded and supervised by the UAE Space Agency, while the probe is being developed by the Mohammed bin Rashid Space Centre (MBRSC), in collaboration with international partners. The objectives of the mission include building Emirati human resources that are highly qualified in space technology, developing knowledge, scientific research and space aconomy, promoting diversification and encouraging innovation, elevating the UAE's status in the space race and further widen benefits. The mission will help to promote the UAE's efforts in scientific explorations, and to build international partnerships in the sector to promote the position of the UAE. For more information, kindly visit Emirates Mars Mission official website through the following link.

Linking over the Emirates Mars Mission page, the teacher and/or students should explore different pages there. Specifically, the teacher should direct students to the 'Vision' tab, scrolling down to the statistics on the value of the space program and its projected yearly growth.

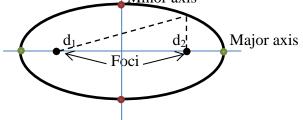
- The teacher should take this opportunity to review exponential functions.
- Exponential growth equations:  $A = P(1 + r)^{t}$ 
  - $\circ$  A = amount at the end
  - P = principle, \$300 billion (according to the website worth noting that this is in dollars, not UAE dirhams, so that students have a proper context for the value)
  - $\circ$  r = rate (as a decimal), 8% (according to the website)
  - t = time (in years) ← the teacher may need to discuss the phrase 'per annum' and it's relation to yearly growth
    - Model:  $A = 300(1 + .08)^{t}$
- What will be the value of the UAE Space Program in 2021, when the Hope Probe arrives at Mars? In 2023, at the potential end of its mission? Or in 2025, at the end of its extended duration?
  - 2021: A = 300(1 + .08)<sup>2</sup> (t = 2, two years past the launch year 2019)
    A = \$349.92 billion
  - 2023: A = 300(1 + .08)<sup>4</sup> (t = 4, four years past the launch year 2019)
    A = \$408.15 billion
  - 2025: A = 300(1 + .08)<sup>6</sup> (t = 6, six years past the launch year 2019)
    A = \$476.06 billion

If the teacher has the capability to do so, he or she should project the image from the Hope Probe page and/or from the Emirates Mars Mission page. The teacher should direct students' attention to the orbit of the probe around Mars. The students should note that the orbit is not circular, but rather *elliptical* (from Emirates Mars Mission 'Journey' tab).



This will allow the teacher to introduce the concept of an ellipse, its characteristics, and how to model it.

- Unlike a circle, with a constant distance from the center to the outside edge (radius), an ellipse does not have such. Rather, an ellipse is a set of points such that the sum of the distances from any point on the ellipse to two other fixed points, called foci (sing: focus), is constant.



In an ellipse,  $d_1 + d_2$  is constant for all points on the ellipse.

- Green dots are called *vertices* (endpoints of the longer axis, or *major* axis) the vertices are  $(h \pm a, k)$  if the major axis is horizontal and  $(h, k \pm a)$  if the major axis is vertical.
- Red dots are called *covertices* (endpoints of the shorter axis, or *minor* axis) the covertices are (h ± b, k) if the major axis is vertical and (h, k ± b) if the major axis is horizontal.
- The center is still defined as (h, k), and the model still maintains something of a circular component.
- The standard equation of an ellipse is  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  (horizontal major axis) or  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$  (vertical major axis), where 2a is the length of the major axis and 2b is the length of the minor axis.
- To calculate the value of the foci, students will use the equation  $c^2 = a^2 b^2$ , or  $c = \pm \sqrt{a^2 b^2}$ , where a > b > 0 (*a* represents major axis and *b* represents minor axis). Because the foci must rest on the major axis, the foci are at  $(h \pm c, k)$  on the horizontal major axis or  $(h, k \pm c)$  on the vertical major axis.

Ex. Give the important characteristics (center, major axis orientation, major axis length, minor axis length, vertices, covertices, foci) of  $\frac{(x-2)^2}{16} + \frac{(y+5)^2}{64} = 1$ . Center: (2, -5) Major Axis is vertical a = 8, so 2a = 16 (length of major axis) Vertices: (2, 3) and (2, -13) b = 4, so 2b = 8 (length of minor axis) Covertices: (6, -5) and (-2, -5)  $c = \pm\sqrt{64 - 16} = \pm\sqrt{48} = \pm 4\sqrt{3} \text{ or } 6.93$ , so foci are  $(2, -5 + 4\sqrt{3})$  and  $(2, -5 - 4\sqrt{3})$ Ex. Give the important characteristics (center, major axis orientation, major axis length, minor axis length, vertices, covertices, foci) of  $\frac{(x+1)^2}{81} + \frac{(y-3)^2}{4} = 1$ . Center: (-1, 3) Major Axis is horizontal a = 9, so 2a = 18 (length of major axis) Vertices: (8, 3) and (-10, 3)

b = 2, so 2b = 4 (length of minor axis)

Covertices: (-1, 1) and (-1, 5)  $c = +\sqrt{81-4} = +\sqrt{77}$ , so vertices are (-1 -  $\sqrt{77}$ ) and (-1 +  $\sqrt{77}$ )

Ex. An ellipse has foci at  $(\pm 6, 3)$  and minor axis of length of 8. What is the equation that models this ellipse?

- If the foci are at  $(\pm 6, 3)$ , then the major axis is horizontal and the center of the ellipse must be at (0, 3), which implies that  $c = \pm 6$ .
- If the minor axis has length 8, then  $2b = 8 \rightarrow b = 4$ .
- $c^2 = a^2 b^2 \rightarrow 6^2 = a^2 4^2 \rightarrow 36 = a^2 16 \rightarrow a^2 = 20$  or  $a = 2\sqrt{5}$  (so Major Axis has length  $4\sqrt{5}$ ).
- Equation:  $\frac{x^2}{20} + \frac{(y-3)^2}{16} = 1$

Ex. If the vertices of an ellipse are (-2, 5) and (-2, -1) and the foci are (-2, 4) and (-2, 0), what is the equation that models this ellipse?

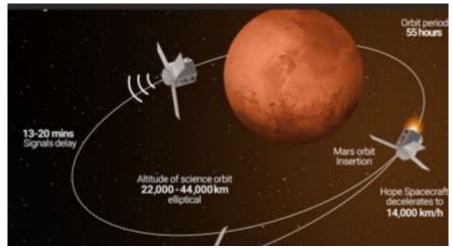
- Center: (-2, 2)
- Because the x-value of the foci does not change but the y-value does, this means that the major axis is oriented vertically, so the *a*-value will be placed under the *y*-term.
- The distance from the center to each vertex is 3, so a = 3.
- If the center is at (-2, 2), the distance to each focus, *c*, is 2, so c = 2.

- 
$$c^{2} = a^{2} - b^{2} \Rightarrow 2^{2} = 3^{2} - b^{2} \Rightarrow b^{2} = 9 - 4 \Rightarrow b^{2} = 5$$

- Equation: 
$$\frac{(x+2)^2}{5} + \frac{(y-2)^2}{9} = 1$$

So how do we model the orbit of the Hope Probe?

- From the Emirates Mars Mission 'Journey' tab, we have this:



- The altitude of the orbit can give the *a* and *b* values, where the major axis would be defined by 44,000 km and the minor axis would be defined by 22,000 km.
- Students would be permitted to define the center (h, k).
  - It might be easiest if they define the center as (0, 0) with an arbitrary coordinate grid, but as long as they can define a center with some mathematical/scientific rationale, that is acceptable.
- Students would need to discuss and agree upon the orientation of the major vs. minor axes.
  - Again, there is not necessarily one 'correct' answer for the orientation, but students should have a rationale justifying their choice of orientation (horizontal or vertical) in writing their equation.
- Horizontal Major Axis:  $\frac{(x-h)^2}{44,000^2} + \frac{(y-k)^2}{22,000^2} = 1$  (Center (*h*, *k*) to be filled in by students)
- Vertical Major Axis:  $\frac{(x-h)^2}{22,000^2} + \frac{(y-k)^2}{44,000^2} = 1$  (Center (h, k) to be filled in by students)

The teacher will take any final questions, both about ellipses and regarding the UAE Space Program. When all questions have been addressed, the teacher will give the day's assignment, which will include an article from *The Guardian* talking about the launch of the Hope Probe, and allow students the remaining time in class to read the article and practice with the mathematical material.

#### Bibliography:

- Agence France-Presse. (2020). "UAE successfully launches Hope probe, Arab world's first mission to Mars." *The Guardian*. <u>https://www.theguardian.com/science/2020/jul/20/uae-mission-mars-al-amal-hope-space</u>.
- Mohammed bin Rashid Space Centre, United Arab Emirates Space Agency. (2015). *Emirates Mars Mission*. Retrieved from <u>http://emiratesmarsmission.ae/mission-journey</u>.
- United Arab Emirates Space Agency. (Feb 19, 2019). UAE Space Agency. Retrieved from <u>https://www.space.gov.ae/</u>.