THE UNIVERSITY OF ARIZONA
COLLEGE OF SOCIAL & BEHAVIORAL SCIENCES
Center for
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# Ellipses: A Lesson for Pre-Calculus Classes

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This lesson will continue to build on a discussion of conic sections, moving away from circles and into ellipses. This discussion will take place via a conversation on the United Arab Emirates Space Agency and its Hope Probe Mars Mission. By introducing the UAE Space Agency and its goals for Mars exploration, students will discuss elliptical orbitals, their shapes, how to model those shapes, and the information that can be gleaned. Students will also be given the opportunity to review exponential growth and how to model it in context.

*Standards*

**North Carolina Pre-Calculus Standard Course of Study**
**(**http://www.ncpublicschools.org/docs/curriculum/mathematics/scos/current/pre-cal.pdf**)**

Competency Goal 1: The learner will describe geometric figures in the coordinate plane geometrically.

*Objective 1.02*: Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.

Competency Goal 2: The learner will use relations and functions to solve problems.

*Objective 2.01*: Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.

a) Solve using graphs and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

*Instruction*

Students will review the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$, as well as the characteristics gleaned from it, namely the center, (h, k), and the radius, r. Students will then be introduced to the United Arab Emirates Space Agency, its history, its goals, and the missions that are currently being planned, with a specific emphasis on the Hope Probe mission.

From our visit with His Excellency Dr. Mohamed Al Junaibi (Executive Director of the Space Sector) and Naser Al Hammadi (Head of Strategic Partnerships Division at the UAE Space Agency), we learned:

- Economic diversification is one of the impetuses of the creation of the UAE Space Agency – 70% non-oil, including space (which is connecting back into alternative energy and aerospace designs)
    - Designed to help create global citizens, centered around all-peace missions
- Why does the UAE want to go to space?

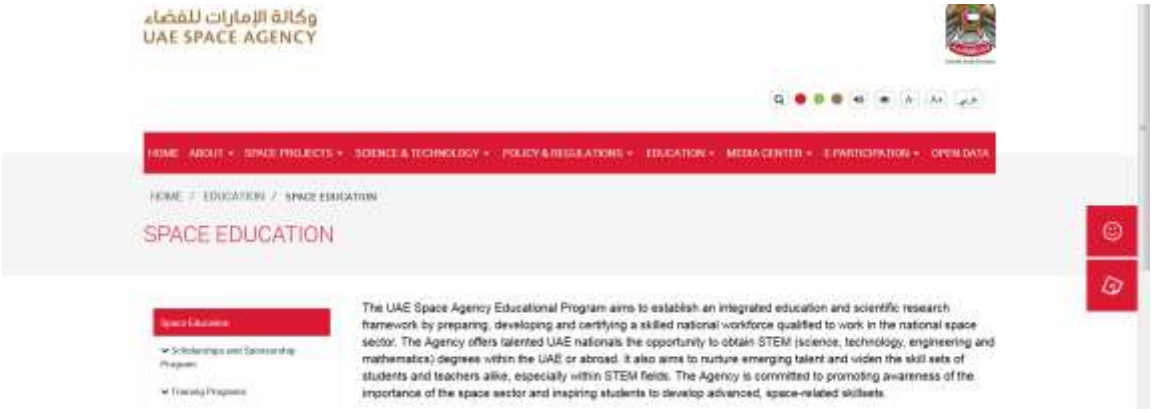
    - Strategic goals: security – in a turbulent region, the UAE is the 2nd strongest economy and aims to be a leader in scientific innovation and bringing stability to the region
    - Scientific goals: ideas and inventions drive innovation, both in the space realm and in aerospace initiatives and advances in technology applicable in daily life
    - Economic goals: to encourage technological innovations and start-ups that can be utilized across economic areas
    - Social goals: navigation and urban planning designs to make lives better
- Aspirations, such as the Mars Mission, drive solutions to challenges that are presented, and those solutions are ten applied to other fields and areas
- History:
    - 1970's – UAE founded, Sheikh Zayed sees Apollo missions and begins to think about how to utilized space
    - 1980's – Universities were started and scientific research began
    - 1990's – Thuraya Telecommunications Company developed in Dubai
    - 2000's – Mohamed Bin Rashid Space Center and Yahsat Satellite Communications Company established
    - 2014 – UAE space agency established
        - It governs the space sector
        - 3 space R&D centers that exceed $6 billion in investment
        - Develop human capital and work with outside agencies and countries, including India, USA, China, Japan, Russia, France, Korea, and Kazakhstan
- The UAE is part of 4 major UN treaties regarding space and are an active member of COPUOUS, ISECG, GEO, The Charter, IPDA, IAF, and more
    - International partnerships and cooperation through treaties with ~20 different countries
- 2020 – will be The Year of Space
    - Mars Mission – 'The Hope' – Want to launch probe to orbit Mars by 2020, and it will arrive there by 2021
    - UAE also has plans for a Mars Colony in 2117
        - Will create Mars Science City in Dubai
        - Focus: education; food, water, and energy challenges (both in UAE and on Mars); labs to include museums and provide entertainment
        - Will be the largest science city in the world (1.9 million $ft^2$)
        - Researchers from all around the world will come to do experiments and do simulations of life on other planets
    - Astronaut program:
        - 2 Emirati citizens currently training to go to the International Space Station in Sept 2019 – one is a fighter pilot and one is a communications engineer with PhD in cyber security
        - Had 4,000 applicants, ~34% of whom were female
            - Were pilots, engineers, doctors, scientists, physicists, and mathematicians – all were considered through interviews, screenings, and physical testing
            - Still want their first female astronaut, hope to select more as space program grows

        - Trained in the US and some in Europe
        - 4 applicants made it through the first rounds of cuts, then these final two were selected and 1 will fly in September – the other will go up in a mission next year
    - Holding internal events
        - "Think Science" – happens every Sunday in an emirate
        - Winter and summer camps within the UAE and in other countries, some of which have been organized with the USA and with Australia
        - Mobile planetariums go to schools
            - Educational awareness is huge, especially with a push in STEM education (12-30% of educational coursework) – working to make STEM more interesting to youth
            - Space camps host astronauts into schools and universities
- Overall, the UAE Space Agency and its work are meant to fulfill the dreams of Sheikh Zayed, develop the UAE as a regional and global leader in space, and be a stabilizing influence in the Middle East

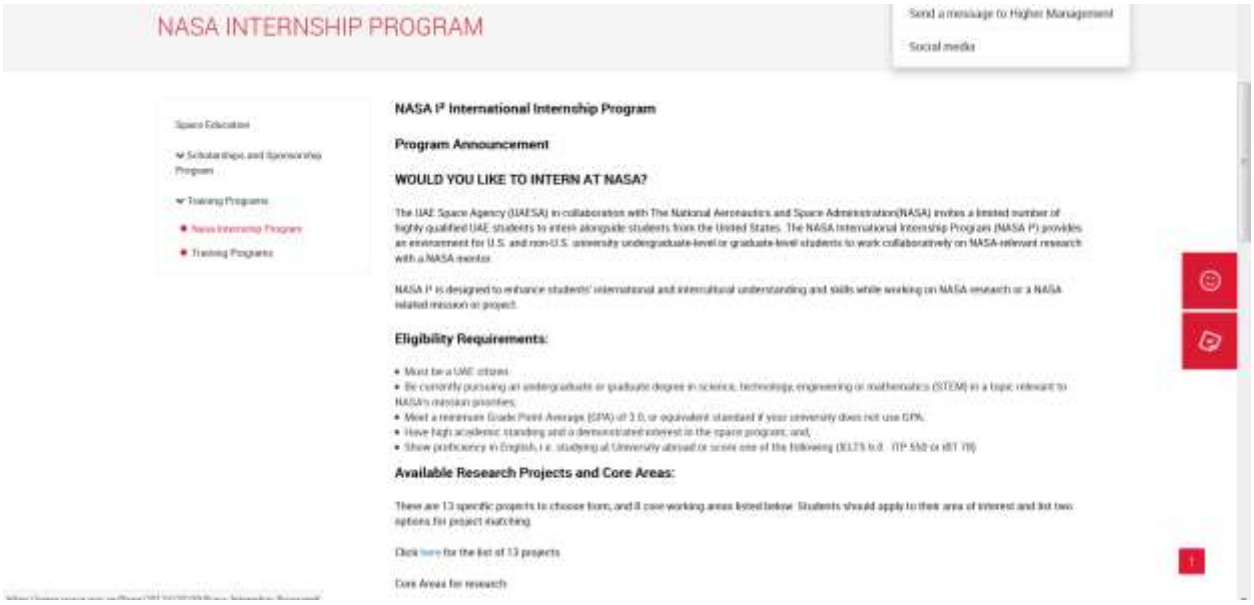
Further information can be found at this website -- https://www.space.gov.ae/.

- Students can either explore this website independently for 10-15 minutes, or the teacher can steer students toward topics of interest.
- Pages of interest:
    - History of the UAE Space Sector https://www.space.gov.ae/Page/20120/20158/History-of-the-UAE-Space-Sector

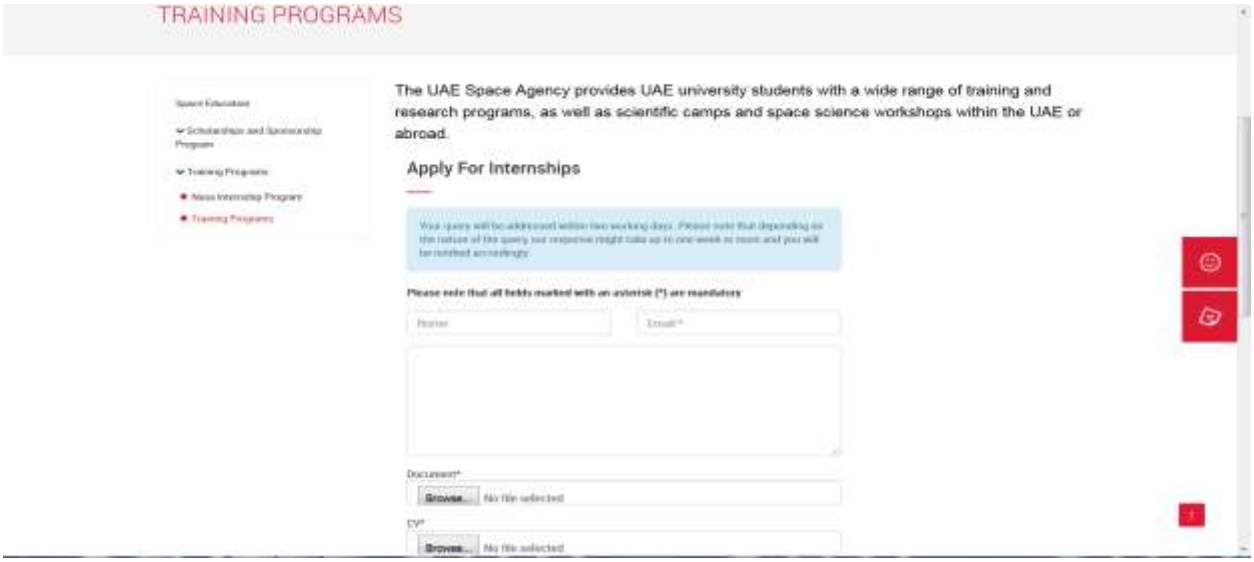

    - Achievements https://www.space.gov.ae/Page/20120/20161/Achievements
    - Space Education https://www.space.gov.ae/Page/20124/20130/Space-Education



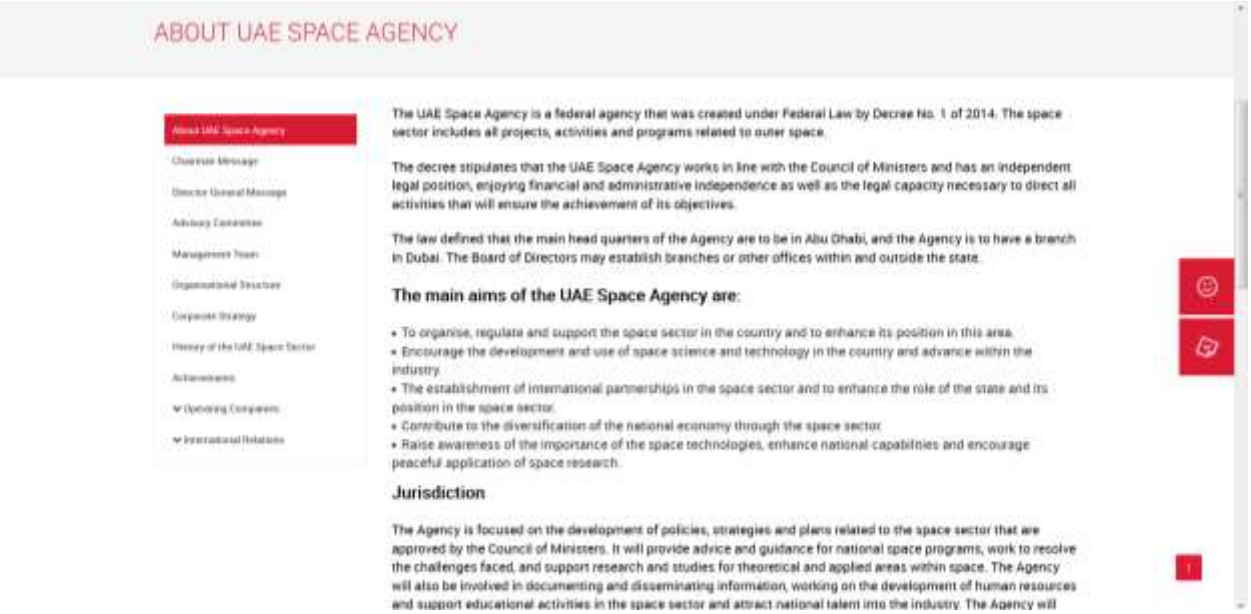
- NASA Internship Program https://www.space.gov.ae/Page/20124/20239/Nasa-Internship-Program



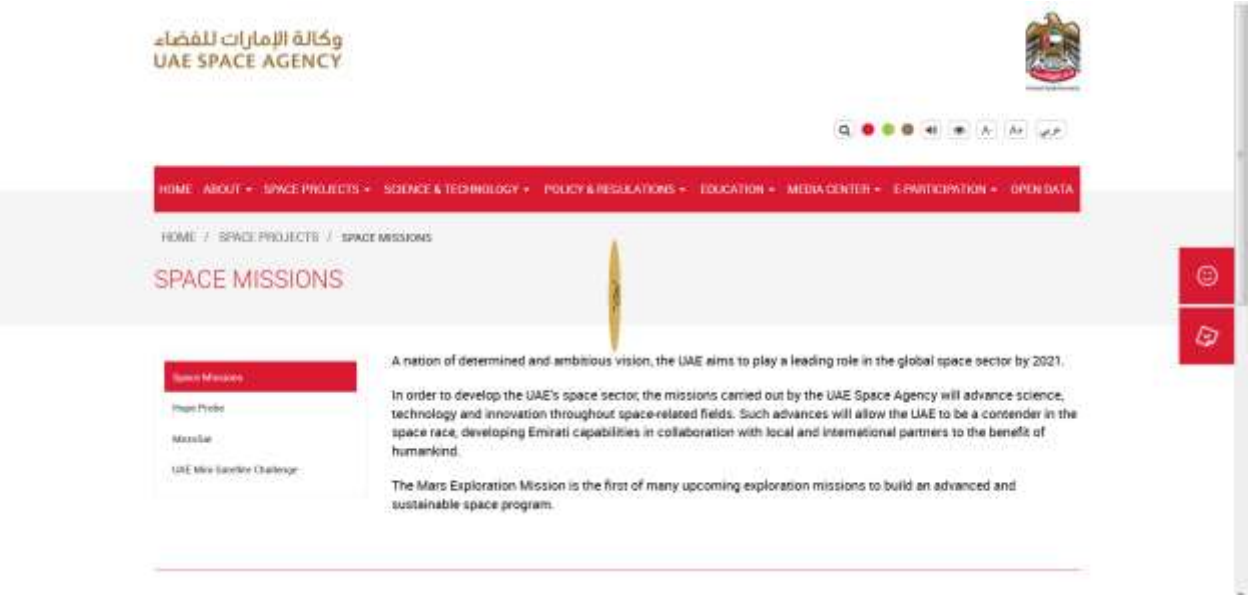

- Training Programs



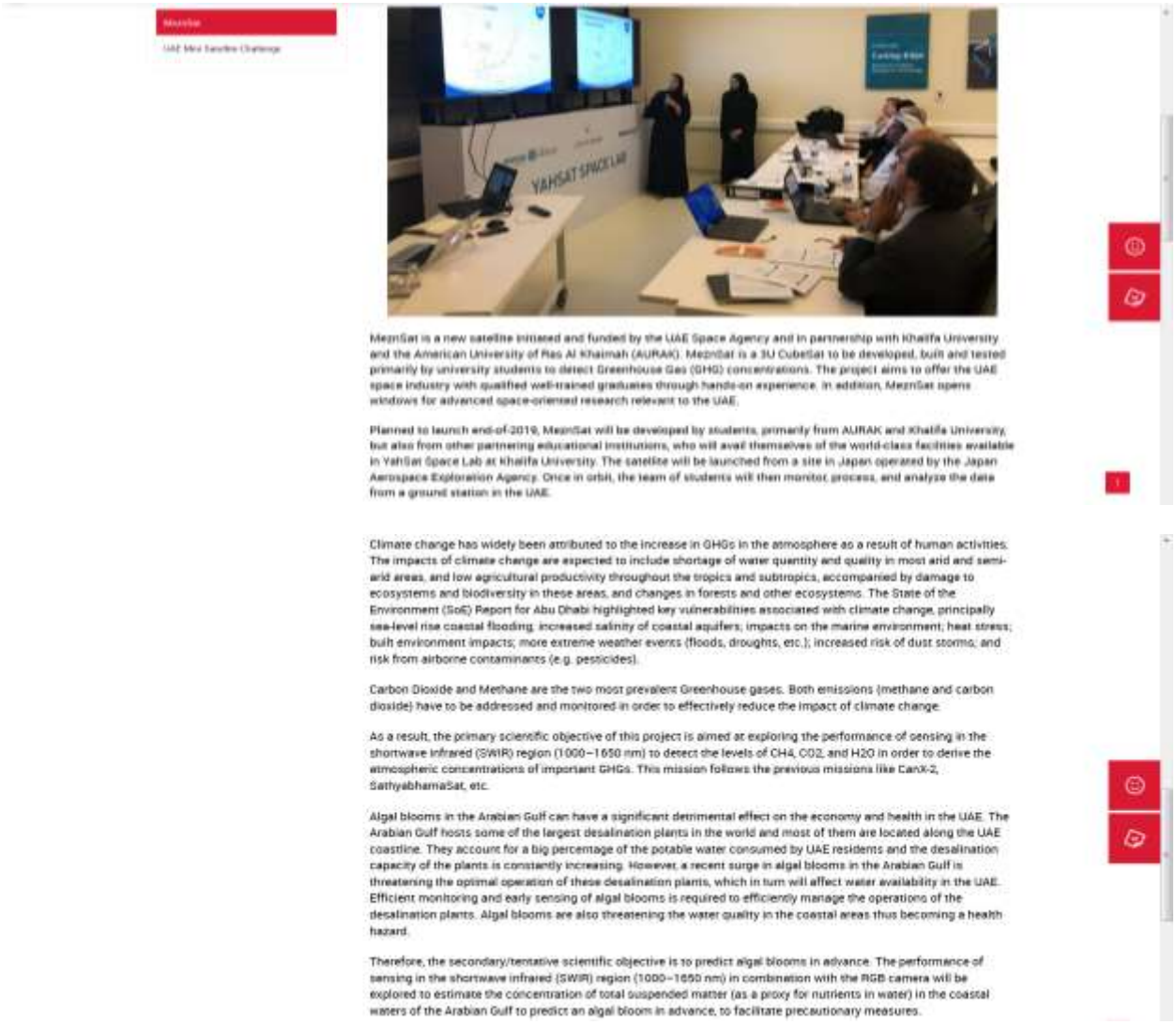

- About UAE Space Agency https://www.space.gov.ae/Page/20120/20230/About-UAE-Space-Agency



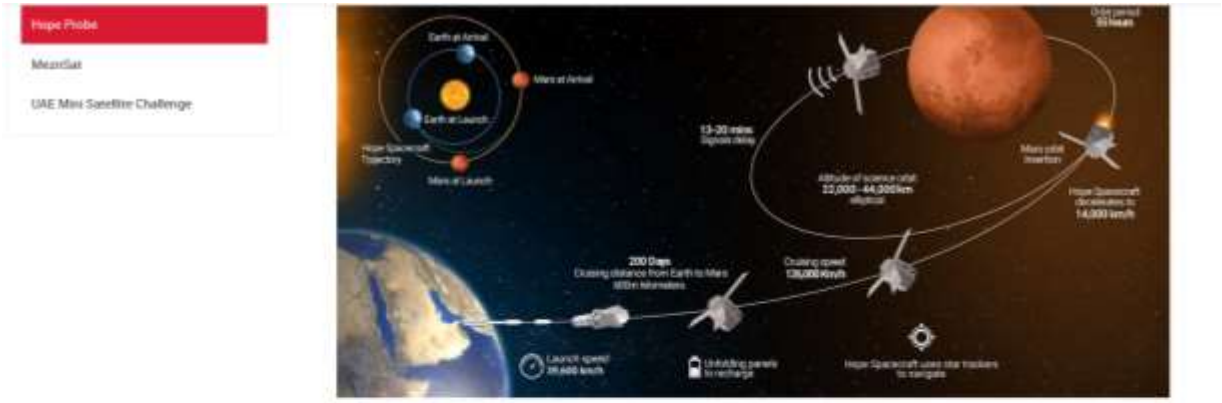

- Space Missions https://www.space.gov.ae/Page/20121/20220/Space-Missions




- MeznSat https://www.space.gov.ae/Page/20121/20223/MeznSat (spend particular time here discussing the impacts of climate change on resource management and scarcity – namely water – in the UAE and the connections of the issues with the Space Agency agenda)




- Hope Probe https://www.space.gov.ae/Page/20121/20167/Hope-Probe (highlight this one last, as it will serve as the anchor of the lesson and introduction to ellipses – utilizing the link at the bottom of the page will redirect to the Emirates Mars Mission page -- http://emiratesmarsmission.ae/mission-journey -- and will provide additional information on the project)

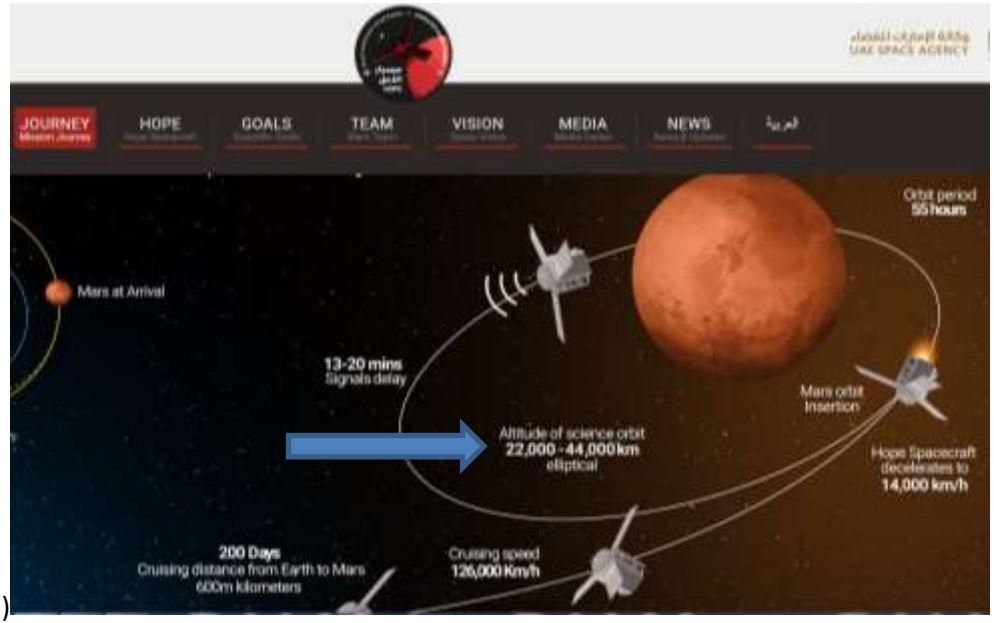



The United Arab Emirates has officially entered the global space exploration race. This came after His Highness Sheikh Khalifa bin Zayed al Nahyan, President of the UAE, issued a decree to establish the UAE Space Agency, and began working on sending the first Arab and Islamic probe to Mars, a project named "Hope Probe". This makes the UAE one of only nine countries working to explore Mars. The probe will start its journey in 2020, and is scheduled to arrive to Mars by 2021, in celebration of the 50th anniversary of the union of the UAE. The planning, management and implementation of the probe project are implemented by an Emirati team, whose members depend on their skills and diligence to acquire all space exploration-related knowledge and applying it. The program is fully funded and supervised by the UAE Space Agency, while the probe is being developed by the Mohammed bin Rashid Space Centre (MBRSC), in collaboration with international partners. The objectives of the mission include building Emirati human resources that are highly qualified in space technology, developing knowledge, scientific research and space applications to benefit mankind. Other goals of the mission include establishing a sustainable knowledge-based economy, promoting diversification and encouraging innovation, elevating the UAE's status in the space race and further widen benefits. The mission will help to promote the UAE's efforts in scientific explorations, and to build international partnerships in the sector to promote the position of the UAE. For more information, kindly visit Emirates Mars Mission official website through the following link.

Linking over the Emirates Mars Mission page, the teacher and/or students should explore different pages there. Specifically, the teacher should direct students to the 'Vision' tab, scrolling down to the statistics on the value of the space program and its projected yearly growth.

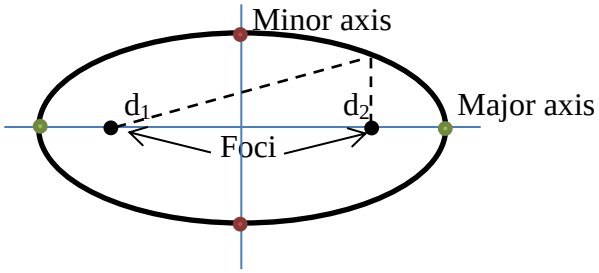
- The teacher should take this opportunity to review exponential functions.
- Exponential growth equations: $A = P(1 + r)^t$
    - A = amount at the end
    - P = principle, $300 billion (according to the website – worth noting that this is in dollars, not UAE dirhams, so that students have a proper context for the value)
    - r = rate (as a decimal), 8% (according to the website)
    - t = time (in years) ← the teacher may need to discuss the phrase 'per annum' and it's relation to yearly growth
        - Model: $A = 300(1 + .08)^t$
- What will be the value of the UAE Space Program in 2021, when the Hope Probe arrives at Mars? In 2023, at the potential end of its mission? Or in 2025, at the end of its extended duration?
    - 2021: $A = 300(1 + .08)^2$ (t = 2, two years past the launch year 2019)
        - A = $349.92 billion
    - 2023: $A = 300(1 + .08)^4$ (t = 4, four years past the launch year 2019)
        - A = $408.15 billion
    - 2025: $A = 300(1 + .08)^6$ (t = 6, six years past the launch year 2019)
        - A = $476.06 billion

If the teacher has the capability to do so, he or she should project the image from the Hope Probe page and/or from the Emirates Mars Mission page. The teacher should direct students' attention to the orbit of the probe around Mars. The students should note that the orbit is not circular, but rather ***elliptical*** (from Emirates Mars Mission 'Journey' tab).




This will allow the teacher to introduce the concept of an ellipse , its characteristics, and how to model it.

- Unlike a circle, with a constant distance from the center to the outside edge (radius), an ellipse does not have such. Rather, an ellipse is a set of points such that the sum of the distances from any point on the ellipse to two other fixed points, called foci (sing: focus), is constant.




  In an ellipse, $d_1 + d_2$ is constant for all points on the ellipse.
- Green dots are called *vertices* (endpoints of the longer axis, or *major* axis) – the vertices are (h ± a, k) if the major axis is horizontal and (h, k ± a) if the major axis is vertical.
- Red dots are called *covertices* (endpoints of the shorter axis, or *minor* axis) – the covertices are (h ± b, k) if the major axis is vertical and (h, k ± b) if the major axis is horizontal.
- The center is still defined as (h, k), and the model still maintains something of a circular component.
- The standard equation of an ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (horizontal major axis) or $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (vertical major axis), where $2a$ is the length of the major axis and $2b$ is the length of the minor axis.

- To calculate the value of the foci, students will use the equation $c^2 = a^2 - b^2$, or $c = \pm\sqrt{a^2 - b^2}$, where $a > b > 0$ ($a$ represents major axis and $b$ represents minor axis). Because the foci must rest on the major axis, the foci are at $(h \pm c, k)$ on the horizontal major axis or $(h, k \pm c)$ on the vertical major axis.

Ex. Give the important characteristics (center, major axis orientation, major axis length, minor axis length, vertices, covertices, foci) of $\frac{(x-2)^2}{16} + \frac{(y+5)^2}{64} = 1$.

Center: (2, -5)
Major Axis is vertical
a = 8, so 2a = 16 (length of major axis)
Vertices: (2, 3) and (2, -13)
b = 4, so 2b = 8 (length of minor axis)
Covertices: (6, -5) and (-2, -5)
$c = \pm\sqrt{64 - 16} = \pm\sqrt{48} = \pm 4\sqrt{3}$ *or* 6.93, so foci are (2, -5 + $4\sqrt{3}$) and (2, -5 – $4\sqrt{3}$)

Ex. Give the important characteristics (center, major axis orientation, major axis length, minor axis length, vertices, covertices, foci) of $\frac{(x+1)^2}{81} + \frac{(y-3)^2}{4} = 1$.

Center: (-1, 3)
Major Axis is horizontal
a = 9, so 2a = 18 (length of major axis)
Vertices: (8, 3) and (-10, 3)
b = 2, so 2b = 4 (length of minor axis)
Covertices: (-1, 1) and (-1, 5)
$c = \pm\sqrt{81 - 4} = \pm\sqrt{77}$, so vertices are (-1 – $\sqrt{77}$) and (-1 + $\sqrt{77}$)

Ex. An ellipse has foci at (±6, 3) and minor axis of length of 8. What is the equation that models this ellipse?

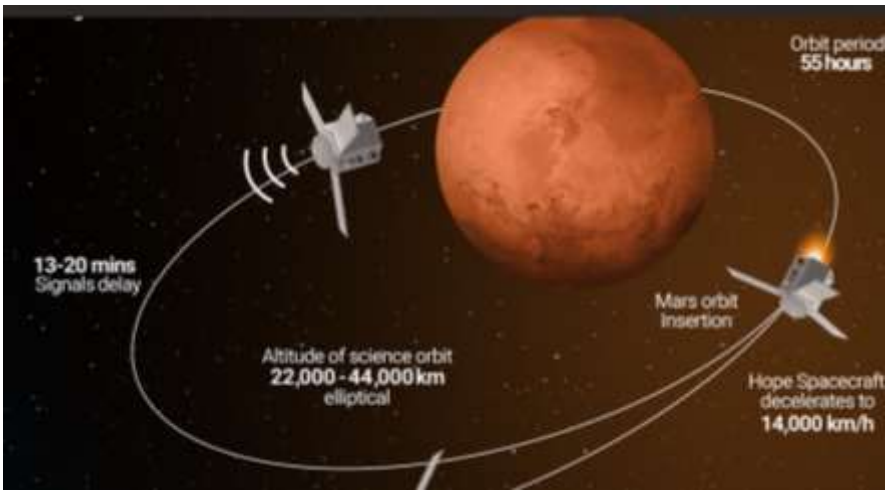
- If the foci are at (±6, 3), then the major axis is horizontal and the center of the ellipse must be at (0, 3), which implies that c = ±6.
- If the minor axis has length 8, then 2b = 8 ➔ b = 4.
- $c^2 = a^2 - b^2$ ➔ $6^2 = a^2 - 4^2$ ➔ $36 = a^2 - 16$ ➔ $a^2 = 20$ or $a = 2\sqrt{5}$ (so Major Axis has length $4\sqrt{5}$).
- Equation: $\frac{x^2}{20} + \frac{(y-3)^2}{16} = 1$

Ex. If the vertices of an ellipse are (-2, 5) and (-2, -1) and the foci are (-2, 4) and (-2, 0), what is the equation that models this ellipse?

- Center: (-2, 2)
- Because the x-value of the foci does not change but the y-value does, this means that the major axis is oriented vertically, so the *a*-value will be placed under the *y*-term.
- The distance from the center to each vertex is 3, so a = 3.
- If the center is at (-2, 2), the distance to each focus, *c*, is 2, so c = 2.
- $c^2 = a^2 - b^2$ ➔ $2^2 = 3^2 - b^2$ ➔ $b^2 = 9 - 4$ ➔ $b^2 = 5$
- Equation: $\frac{(x+2)^2}{5} + \frac{(y-2)^2}{9} = 1$

So how do we model the orbit of the Hope Probe?

- From the Emirates Mars Mission ‘Journey’ tab, we have this:




- The altitude of the orbit can give the *a*- and *b* values, where the major axis would be defined by 44,000 km and the minor axis would be defined by 22,000 km.
- Students would be permitted to define the center (h, k).
    - It might be easiest if they define the center as (0, 0) with an arbitrary coordinate grid, but as long as they can define a center with some mathematical/scientific rationale, that is acceptable.
- Students would need to discuss and agree upon the orientation of the major vs. minor axes.
    - Again, there is not necessarily one 'correct' answer for the orientation, but students should have a rationale justifying their choice of orientation (horizontal or vertical) in writing their equation.
- Horizontal Major Axis: $\frac{(x-h)^2}{44{,}000^2} + \frac{(y-k)^2}{22{,}000^2} = 1$ (Center ($h$, $k$) to be filled in by students)
- Vertical Major Axis: $\frac{(x-h)^2}{22{,}000^2} + \frac{(y-k)^2}{44{,}000^2} = 1$ (Center ($h$, $k$) to be filled in by students)

The teacher will take any final questions, both about ellipses and regarding the UAE Space Program. When all questions have been addressed, the teacher will give the day's assignment, which will include an article from *The Guardian* talking about the launch of the Hope Probe, and allow students the remaining time in class to read the article and practice with the mathematical material.

*Bibliography*:

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